A hybrid controller for obstacle avoidance in an n-dimensional Euclidean space

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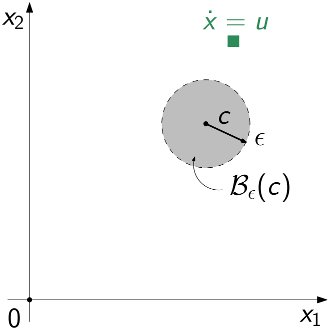
Joint work with Andrea Bisoffi and Dimos V. Dimarogonas

A classical problem

- Vehicle in $\mathbb{R}^n$ ($n \geq 2$) with state $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^n$ and single integrator dynamics:

$$\dot{x} = u$$

- **Obstacle** is a spherical region $\mathcal{B}_\epsilon(c)$ with center $c \in \mathbb{R}^n$, radius $\epsilon > 0$.
- Avoid the obstacle while stabilizing the vehicle position to a given reference, the origin $x = 0$.



Outline

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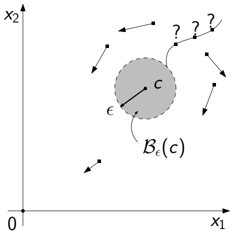
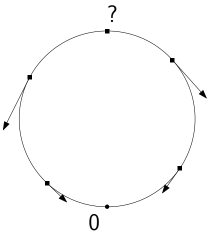
1. Overview
2. Solution: hybrid system construction
3. Solution: main result
4. Discussion & extensions

The topological obstruction[1,2]

- The origin can **not** be **globally** asymptotically stabilized by **continuous** pure state feedback in the presence of the obstacle.



similar problem for dynamical systems on spheres



- Even **discontinuous** pure state feedback cannot achieve global asymptotic stability that is **robust** to **arbitrarily small** measurement noise.

[1] D. Liberzon. *Switching in systems and control*, Springer, 2003.

[2] R. G. Sanfelice, M. J. Messina, S. E. Tuna, and A. R. Teel. Robust hybrid controllers for continuous-time systems with applications to obstacle avoidance and regulation to disconnected sets of points. ACC, 2006.

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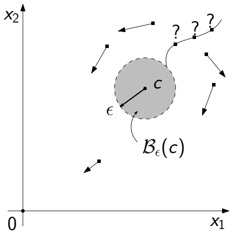

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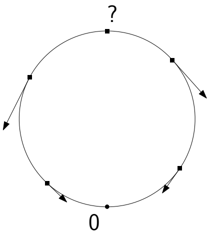

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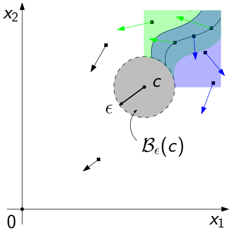
Motivation to use hybrid feedback

- Here we use **hysteresis switching** (proposed, e.g., in [3]) as hybrid mechanism to guarantee robustness.



[3] R. G. Sanfelice, M. J. Messina, S. E. Tuna, and A. R. Teel. Robust hybrid controllers for continuous-time systems with applications to obstacle avoidance and regulation to disconnected sets of points. ACC, 2006.

(Partial) literature survey

- **Artificial potential fields**[4]:
  - pioneering work within the robotics community;
  - render the origin attractive and the obstacles repulsive;
  - the vehicle navigates along the negative gradient of the artificial potential field;
  - the potential has local minima and is arbitrarily large near the obstacles.

[4] O. Khatib. Real-time obstacle avoidance for manipulators and mobile robots. Autonomous robot vehicles, 1986.

(Partial) literature survey

- **Artificial potential fields**
- **Navigation functions for spherical worlds**[4]:
  - all critical points of the artificial potential fields are saddles expect the origin;
  - extended in many directions (multi-agent systems, unknown sphere words, etc.);
  - they are theoretically guaranteed to exist, but they require a parameter to be arbitrarily large to eliminate local minima.

[4] D. E. Koditschek and E. Rimon. Robot navigation functions on manifolds with boundary. Advances in applied mathematics, 1990.

(Partial) literature survey

- **Artificial potential fields**
- **Navigation functions**
- **Control barrier functions**[4]:
  - control **Lyapunov** functions are (Lyapunov) functions that can be made decrease by applying a suitable control, so that stabilization of the origin is achieved;
  - control **barrier** functions works similarly to guarantee the invariance of a safe set.

[4] P. Wieland and F. Allgöwer. Constructive safety using control barrier functions. IFAC NOLCOS, 2007.
M. Z. Romdlony and B. Jayawardhana. Stabilization with guaranteed safety using control Lyapunov-barrier function. Automatica, 2016.
A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada. Control barrier function based quadratic programs for safety critical systems. IEEE Trans. Automat. Contr., 2017.

(Partial) literature survey

- **Artificial potential fields**
- **Navigation functions**
- **Control barrier functions**
- **Hybrid systems solutions**:
  - [4] and [5] consider a planar setting ($\mathbb{R}^2$) ;
  - [6] consider a class of linear systems in $\mathbb{R}^n$ and an unsafe point as obstacle.
  - Using tools from higher dimensional geometry, we propose a hybrid scheme for $\mathbb{R}^n$ (can be extended to multiple obtacles)

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[4] R. G. Sanfelice, M. J. Messina, S. E. Tuna, and A. R. Teel. Robust hybrid controllers for continuous-time systems with applications to obstacle avoidance and regulation to disconnected set of points. ACC, 2006.

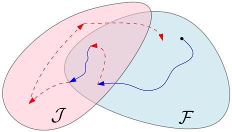
[5] J. I. Poveda, M. Benosman, A. R. Teel, and R. G. Sanfelice. A hybrid adaptive feedback law for robust obstacle avoidance and coordination in multiple vehicle systems. ACC, 2018.

[6] P. Braun, C. M. Kellett, and L. Zaccarian. Unsafe point avoidance in linear state feedback. CDC, 2018.

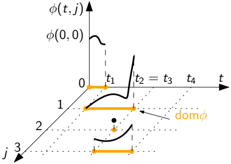
Essential ingredient: hybrid systems[7]

- State equations:

$$\begin{cases} \dot{x} \in \mathbf{F}(x), & x \in \mathcal{F} \\ x^+ \in \mathbf{J}(x), & x \in \mathcal{J} \end{cases}$$



- Parametrize solut's by elapsed continuous time t and number of discrete jumps j
- *Hybrid solutions* $(t, j) \mapsto \phi(t, j)$ defined on their *hybrid time domain* dom ϕ



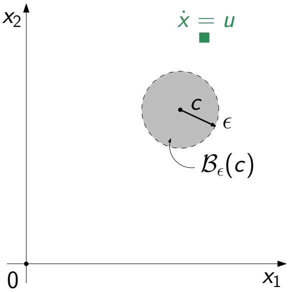
[7] R. Goebel, R. G. Sanfelice, and A. R. Teel, *Hybrid dynamical systems: modeling, stability, and robustness*. Princeton University Press 2012.

Outline

Objectives

Objectives for the design of control input u

1. **Safety:** the obstacle-free region $\mathbb{R}^n \setminus \mathcal{B}_\epsilon(c)$ is forward invariant;
2. **Stabilization:** the origin $x = 0$ is globally asymptotically stable;
3. **Preservation:** for each $\epsilon' > \epsilon$, there exist controller parameters such that the control law matches, in $\mathbb{R}^n \setminus \mathcal{B}_{\epsilon'}(c)$, the law $u = -k_0 x$ ($k_0 > 0$) used in the absence of the obstacle.



- 1 and 2 cannot be both achieved by continuous feedback due to the topological obstruction;
- 3 [8] is desirable when the controller modifications imposed by the presence of the obstacle should be as minimal as possible.

[8] P. Braun, C. M. Kellett and L. Zaccarian, *Unsafe point avoidance in linear state feedback*, IEEE Conf. Dec. Contr., 2018.

Control law

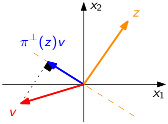
- The control law u has three **modes** $m \in \{-1, 0, 1\}$
- Stabilization mode $m = 0$

$$u = \kappa(x, 0) = -k_0 x, \quad k_0 > 0.$$

- Avoidance mode $m \in \{-1, 1\}$

$$u = \kappa(x, m) = \pi^{\perp}(x - c)(-k_m(x - p_m)), \quad k_m > 0.$$

- $\pi^{\perp}(z)v$: projection of v onto the hyperplane orthogonal to z.



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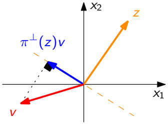

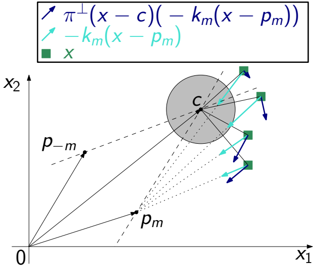

Unsafe region for stabilization ($u = -k_0 x$)

- Idea: **do not** flow when x is **close** to the obstacle and $-k_0 x$ **points towards** the obstacle.

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$$\epsilon \leq \|x - c\| \leq \epsilon_s$$
$$\frac{\mathrm{d}}{\mathrm{dt}}\|x - c\|^2 = -2k_0 x^\top (x - c) \leq 0$$

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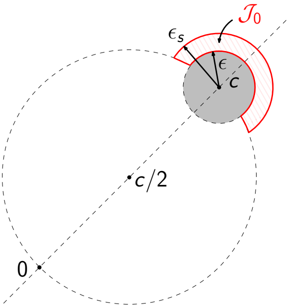
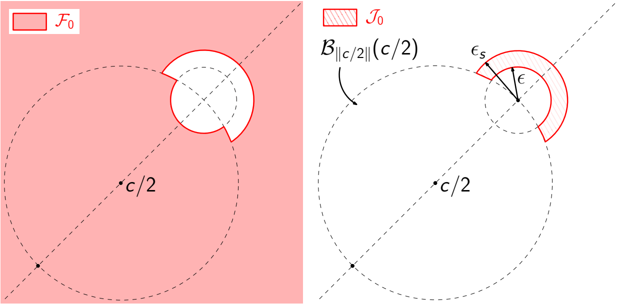
- If the vehicle hits the safety helmet $\mathcal{J}_0$ during stabilization, then the system needs to jump to avoidance.



Flow and jump set for stabilization

- This leads to the selection of flow and jump sets for avoidance.



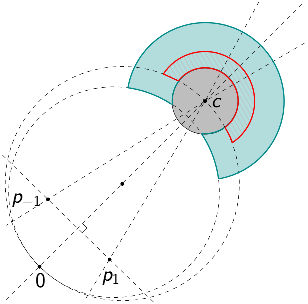
- Their union covers $\mathbb{R}^n \setminus \mathcal{B}_\epsilon(c)$.

Flow and jump sets for avoidance: (undesired) equilibria

- The control law for avoidance was

$$u = \kappa(x, m) := -k_m \pi^{\perp}(x - c)(x - p_m), \quad m \in \{-1, 1\} \qquad (\spadesuit)$$

- Take an inflated safety helmet as **preliminary** flow set for avoidance



- The hysteresis introduced by the inflation avoids Zeno behaviour (chattering between modes).
- Equilibria of the flow map for avoidance in ($\spadesuit$):

$$\pi^{\perp}(x - c)(x - p_m) = 0 \iff x \in \mathcal{L}(c, p_m).$$

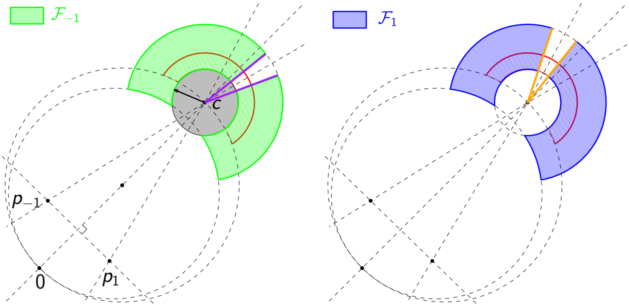
Flow and jump sets for avoidance: selection of flow sets

- We then select the point p_m and the flow set $\mathcal{F}_m$ such that

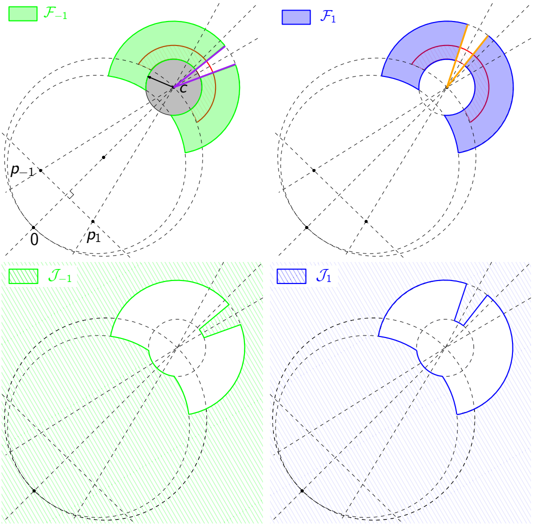
$$\mathcal{L}(c, p_m) \cap \mathcal{F}_m = \emptyset, \quad m \in \{-1, 1\},$$

otherwise solutions can stay in the avoidance mode indefinitely.

- We achieve this by taking intersection with suitable conic regions.



Flow and jump sets for avoidance: selection of jump sets

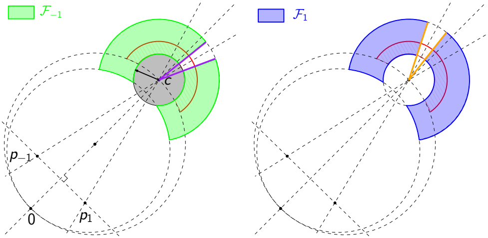


Jump map

- The jump map is

$$\begin{cases} x^+ = x \\ m^+ \in \mathbf{M}(x, m) \end{cases} \qquad (x, m) \in \bigcup_{m \in \{-1,0,1\}} \mathcal{J}_m \times \{m\}.$$

- The **jump map** specifies precisely **how we select which mode** through **M**.



- $\mathbf{M}(x, 0) \subseteq \{-1, 1\}$
- $\mathbf{M}(x, \{-1, 1\}) := \{0\}$

Overall hybrid system

- We bring all the previous elements together:

$$\begin{cases} \dot{x} = \kappa(x, m) \\ \dot{m} = 0 \end{cases}$$

$$(x, m) \in \bigcup_{m \in \{-1,0,1\}} \mathcal{F}_m \times \{m\}$$

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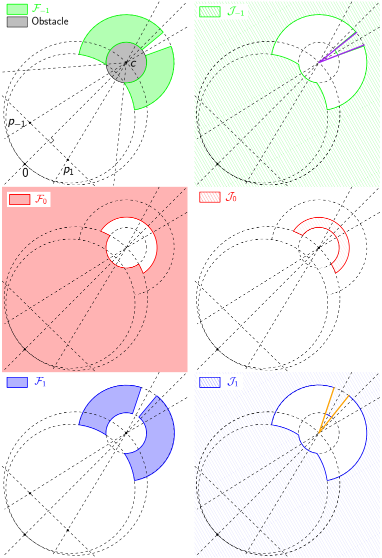


Outline

Main result

- Define the obstacle-free set $\mathcal{K}$ and the attractor $\mathcal{A}$

$$\mathcal{K} := \overline{\mathbb{R}^n \setminus \mathcal{B}_\epsilon(c)} \times \{-1, 0, 1\}, \quad \mathcal{A} := \{0\} \times \{0\}.$$

Main result

For the constructed hybrid system under a suitable parameter selection,

i) all (maximal) solutions are complete in the direction of time t, and the obstacle-free set $\mathcal{K}$ is forward invariant;

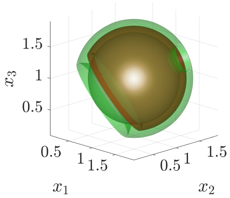
ii) the set $\mathcal{A}$ is globally asymptotically stable;

iii) $\forall \epsilon' > \epsilon$, $\exists$ controller parameters so that the resulting hybrid feedback law matches, in $\mathbb{R}^n \setminus \mathcal{B}_{\epsilon'}(c)$, the nominal law $u = -k_0 x$.

Simulations: one spherical obstacle and $n = 3$

- Relevant sets for $n = 3$:

$\mathcal{F}_{-1}$, $\mathcal{J}_0$, $\mathcal{B}_\epsilon(c)$



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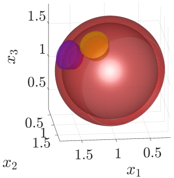

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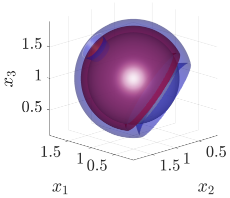

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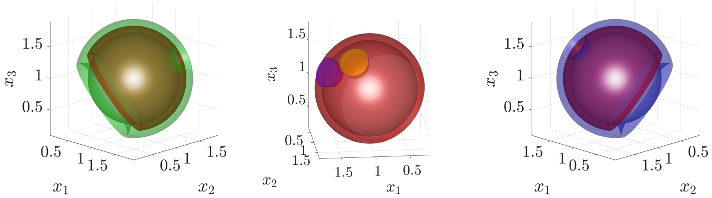
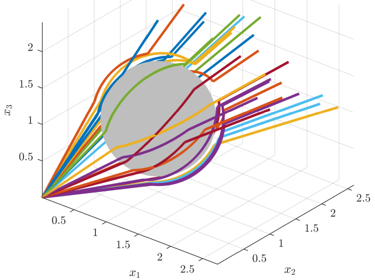


- **Stabilization** and **preservation**: $x = 0$ is globally asymptotically stable, and the control law u matches $-k_0 x$ sufficiently away from the obstacle.



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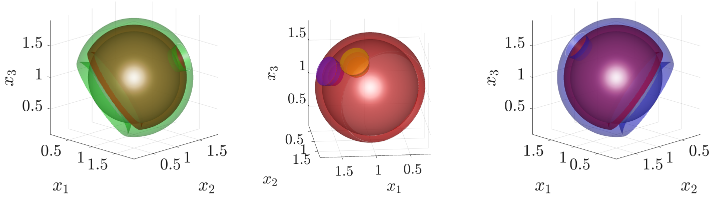
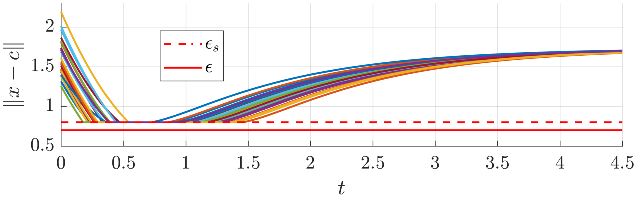
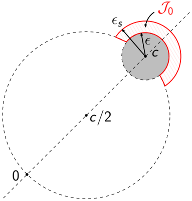


- **Safety**: distance to the obstacle $\|x - c\|$, radii ϵ_s, ϵ





Extension to multiple ellipsoidal obstacles

- Thanks to the **preservation** property, the approach is modular and can be extended to multiple obstacles.

Conclusions

- We have proposed the construction of a hybrid system in a n-dimensional Euclidean space for stabilization in the presence of a spherical obstacle.
- The resulting closed-loop hybrid systems enjoys the properties of safety, (robust) global asymptotic stability, and preservation.
- This construction seems promising to be extended to more interesting surfaces.

Thanks for your attention!