Position, Velocity, Attitude and Gyro-Bias Estimation from IMU and Position Information

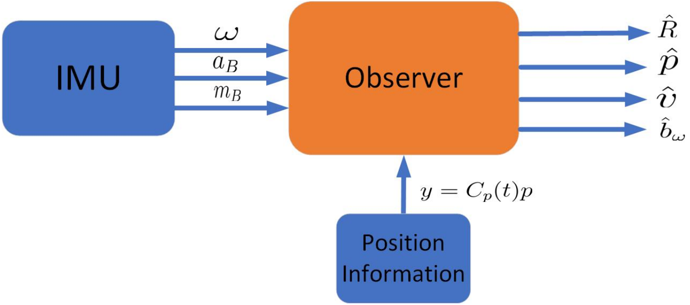
Soulaimane Berkane [1] Abdelhamid Tayebi [2]

[1]KTH Royal Institute of Technology

[2]Lakehead University

June 28, 2019

Problem Formulation



$$\dot{p} = v,$$
$$\dot{v} = g + Ra_B,$$
$$\dot{R} = R[\omega]_\times,$$

- Apparent acceleration $a_I = Ra_B$ is unknown $\rightarrow$ we need to estimate it as well for use in the attitude observer

Translational Dynamics

Let $x := (p, v, a_I) \in \mathbb{R}^9$.

$$\dot{x} = Ax + B_1 g + B_2 \dot{a}_I$$
$$y = C(t)x$$

$$A = \begin{bmatrix} 0 & I & 0 \\ 0 & 0 & I \\ 0 & 0 & 0 \end{bmatrix}, B_1 := \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix}, B_2 := \begin{bmatrix} 0 \\ 0 \\ I \end{bmatrix}, \quad C(t) = \begin{bmatrix} C_p(t) \\ 0 \\ 0 \end{bmatrix}^\top$$

- Linear system with unknown input $\dot{a}_I$
- We only measure $a_B = R^\top a_I$

Translational Motion Observer

Consider the "quasi-linear" observer

$$\begin{cases} \hat{x} & = \hat{z} + B_2 \hat{R} a_B \\ \dot{\hat{z}} & = A\hat{x} + B_1 g + K(t)(y - C(t)\hat{x}) + \sigma_1(\hat{R}) \end{cases}$$

with $\sigma_1(\hat{R}) \to 0$ when $\hat{R} \to R$ (note that $\hat{R} a_B \to a_I$).

- If $\hat{R} \equiv R$, this is equivalent to the **Luenberger-type observer**

$$\dot{\hat{x}} = A\hat{x} + B_1 g + B_2 \dot{a}_I + K(t)(y - C(t)\hat{x}).$$

- $K(t) = \gamma L_\gamma P(t) C(t)^\top Q(t)$, with $\gamma \geq 1$,
  $L_\gamma = \mathrm{blockdiag}(I_3, \gamma I_3, \gamma^2 I_3)$ and $P(t)$ is solution of the CRE

$$\frac{1}{\gamma}\dot{P} = AP + PA^\top - PC(t)^\top Q(t) C(t) P + V(t),$$

Rotational Motion Observer

Consider the "nonlinear complementary filter"-type observer[1]

$$\begin{cases} \dot{\hat{R}} & = \hat{R}[\omega^y - \hat{b}_\omega + k_1\sigma_2]_\times \\ \dot{\hat{b}}_\omega & = \mathbf{Proj}(\hat{b}_\omega, -k_2\sigma_2) \end{cases}$$

$$\sigma_2 = \rho_1(m_B \times \hat{R}^\top m_I) + \rho_2(a_B \times \hat{R}^\top \mathbf{sat}(\hat{a}_I))$$

- **Proj**: is the parameter projection function
- **sat**: is a saturation function

[1] R. Mahony, T. Hamel and J. Pflimlin, "**Nonlinear Complementary Filters on the Special Orthogonal Group**," in IEEE TAC, 2008.
H. F. Grip, T. I. Fossen, T. A. Johansen and A. Saberi, "**Attitude Estimation Using Biased Gyro and Vector Measurements With Time-Varying Reference Vectors**," in IEEE TAC, 2012.

Main Result

Assumption

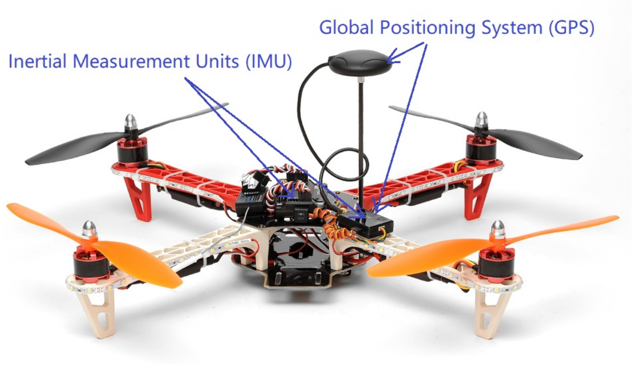
- *Pair* $(A, C(\cdot))$ *is uniformly observable*
- m_I *and* $a_I(t)$ *are not collinear for all times*
- $\omega, b_\omega, a_I, \dot{a}_I$ *are uniformly bounded*

Theorem (Semi-global exponential stability)

For all initial conditions (except attitude errors at 180°*), there exist (high) gains such that the estimation error converges exponentially to zero.*

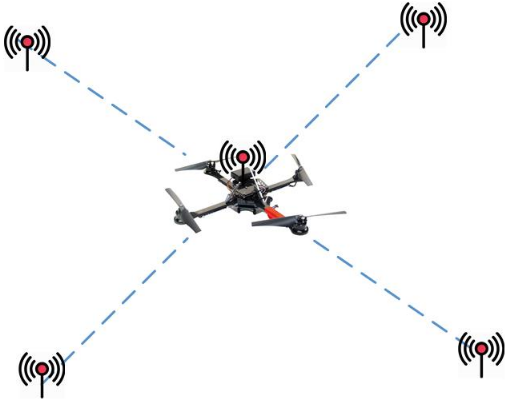
- Although, sufficiently large gains are needed in the proof, simulation results demonstrate that this is very conservative.

Full position measurement in the inertial frame (e.g., GPS)



$$C_p = I_{3\times 3}$$

Range measurements (e.g., Ultra-wide-Band (UWB))



Range measurements (e.g., Ultra-wide-Band (UWB))

At least 4 non-coplanar source points are needed located at p_i.

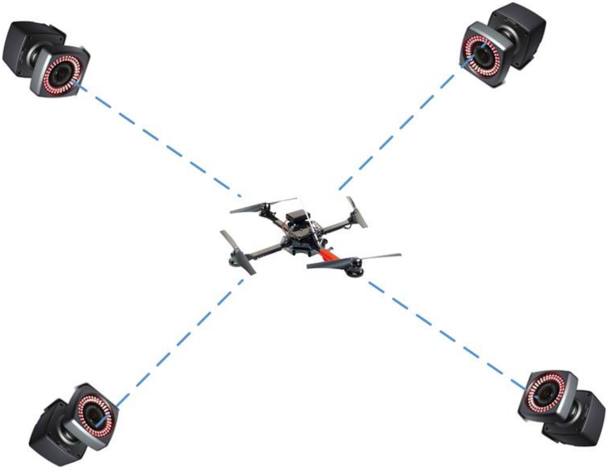
$$d_i = \|p - p_i\|, \quad i = 1, \cdots, n.$$

Output equaion:

$$y_i := \frac{1}{2}\left(d_i^2 - d_1^2 - \|p_i\|^2 + \|p_1\|^2\right), \quad i = 2, \cdots, n.$$

$$y = \begin{bmatrix} (p_1 - p_2)^\top \\ \vdots \\ (p_1 - p_n)^\top \end{bmatrix} p := C_p p \tag{1}$$

Bearing measurements in the inertial frame



Bearing measurements in the inertial frame

$$b_i = R_i^\top \frac{p - p_i}{\|p - p_i\|}, \quad i = 1, \cdots, n,$$

- $R_i \in \mathbb{SO}(3)$: Orientation of camera i w.r.t. the inertial frame.
- p_i: Position of camera i in the inertial frame.
- Note that

  $$\Pi(b_i) R_i^\top (p - p_i) = 0$$

  where $\Pi(z) := I - \frac{zz^\top}{\|z\|^2}$ is the orthogonal projection.
- Therefore,

  $$y_i := \Pi(b_i) R_i^\top p_i = \Pi(b_i) R_i^\top p$$

  and letting $y = (y_1, \cdots, y_n)$ we have

  $$y = \begin{bmatrix} \Pi(b_1) R_1^\top \\ \vdots \\ \Pi(b_n) R_n^\top \end{bmatrix} p := C_p(t) p.$$

Bearing measurements

Lemma (Uniform observability)

$(A, C(t))$ *is uniformly observable if there exist* $\delta, \mu > 0$ *such that for all* $t \geq 0$ *one has*

$$\frac{1}{\delta}\int_t^{t+\delta} \sum_{i=1}^{n} \Pi(R_i b_i(s)) ds \geq \mu I_3.$$

- **Single camera:** Vehicle is never static nor moving on a straight line passing through the camera.
- **Two cameras:** Vehicle not aligned (indefinitely) with both cameras.
- **Three cameras or more:** If at least 3 cameras are not aligned, the observability condition is always satisfied.

Bearing measurements

Lemma (Uniform observability)

$(A, C(t))$ is uniformly observable if there exist $\delta, \mu > 0$ such that for all $t \geq 0$ one has

$$\frac{1}{\delta}\int_t^{t+\delta}\sum_{i=1}^{n}\Pi(R_i b_i(s))ds \geq \mu I_3.$$

- **Single camera:** Vehicle is never static nor moving on a straight line passing through the camera.
- **Two cameras:** Vehicle not aligned (indefinitely) with both cameras.
- **Three cameras or more:** If at least 3 cameras are not aligned, the observability condition is always satisfied.

Bearing measurements

Lemma (Uniform observability)

$(A, C(t))$ *is uniformly observable if there exist* $\delta, \mu > 0$ *such that for all* $t \geq 0$ *one has*

$$\frac{1}{\delta} \int_t^{t+\delta} \sum_{i=1}^{n} \Pi(R_i b_i(s)) ds \geq \mu I_3.$$

- **Single camera:** Vehicle is never static nor moving on a straight line passing through the camera.
- **Two cameras:** Vehicle not aligned (indefinitely) with both cameras.
- **Three cameras or more:** If at least 3 cameras are not aligned, the observability condition is always satisfied.

Altimeter

- An altimeter is a sensor that measures the altitude

$$y = [0\ 0\ 1]p.$$

Lemma (Uniform observability: bearings+altimeter)

$(A, C(t))$ is uniformly observable if there exist $\delta, \mu > 0$ such that for all $t \geq 0$ one has

$$\frac{1}{\delta}\int_{t}^{t+\delta}\sum_{i=1}^{n}\Pi(R_i b_i(s))ds + e_3 e_3^\top \geq \mu I_3.$$

- **Single camera + altimeter**: Vehicle not moving at the same altitude as the camera or, if doing so, not moving on a straight line passing through the camera
- **Two cameras + altimeter**: Cameras not at the same altitude. If the cameras are at the same altitude, vehicle must not be aligned (indefinitely) with both cameras.

Simulations: true trajectory

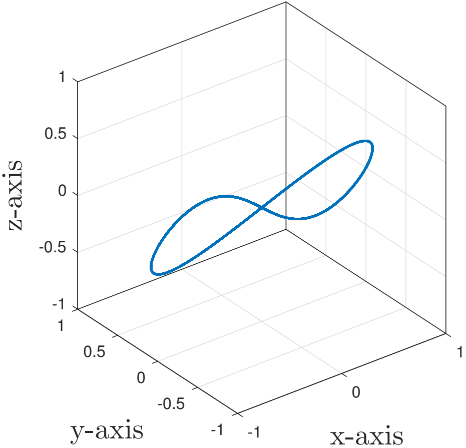


Figure 1: True trajectory of the vehicle.

Simulations: single bearing

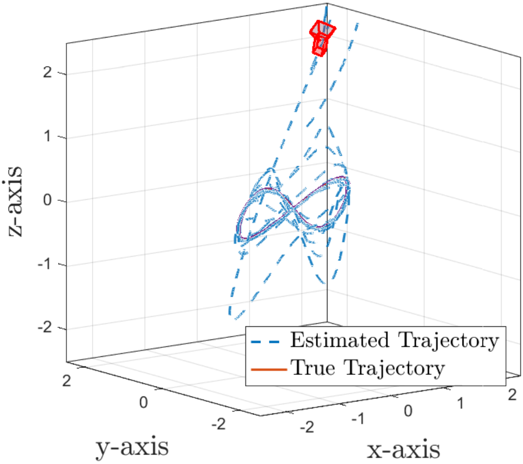


Figure 2: Observability holds but slow convergence.

Simulations: single bearing + altimeter

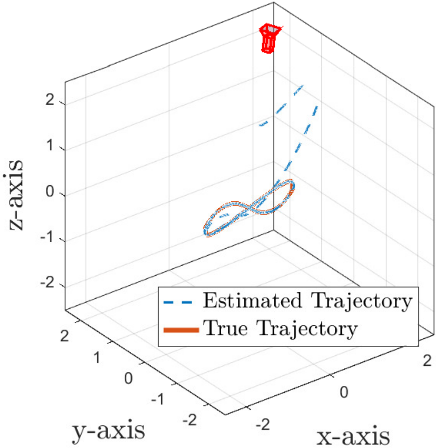


Figure 3: Stronger observability.

Conclusion

- A **generic** navigation observer with semi-global exponential stability
- Position information (full, bearings, range) in the inertial frame
- Future work using information in the body-frame (e.g., on-board cameras)

Conclusion

- A **generic** navigation observer with semi-global exponential stability
- Position information (full, bearings, range) in the inertial frame
- Future work using information in the body-frame (e.g., on-board cameras)

Conclusion

- A **generic** navigation observer with semi-global exponential stability
- Position information (full, bearings, range) in the inertial frame
- Future work using information in the body-frame (e.g., on-board cameras)

Thank you

Questions?