Hybrid Control and Estimation for Systems with Constraints

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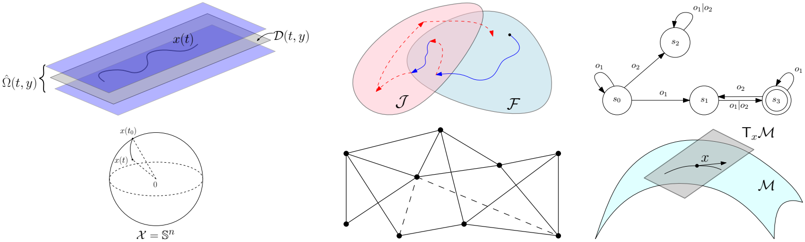
Introduction

Academic Career

- **Ecole National Polytechnique** (ENP), Algeria
  - **Engineer** (Automation & Control) (2008-2013)
  - **M.Sc.** (Automatic Control) (2011-2013)
- **University of Western Ontario** (UWO), Canada
  - **Ph.D.** (Robotics & Control) (2014-2017)
  - **Post-Doc** (Prof. A. Tayebi) (7 months, 2018)
- **KTH Royal Institute of Technology**, Sweden
  - **Post-Doc** (Prof. D. V. Dimarogonas) (2018-Current)

Introduction

Research Interests (Theory)



- Hybrid Dynamical Systems
- Systems with Constraints
- Multiagent Systems
- Formal Verification Methods

Introduction

Research Interests (Applications)








- Navigation and Control of Unmanned Vehicles
- Cooperative Localization for Autonomous Vehicles
- Multi-Robot Systems Coordination & Task Planning

Outline

Systems with Constraints

Topological Incompatibility

Hybrid Systems Framework

Control Examples

Estimation Examples

Systems with Constraints

Consider the system dynamics

$$\dot{x} = f(x, u) \qquad x \in \mathcal{X}, \quad u \in \mathcal{U}$$
$$y = h(x) \qquad y \in \mathcal{Y}$$

- state-space constraints

$$\mathcal{X} \subset \mathbb{R}^n$$

- Input constraints

$$\mathcal{U} \subset \mathbb{R}^n$$

- Output constraints

$$\mathcal{Y} \subset \mathbb{R}^n$$

- Time constraints (*e.g.*,delay, discrete-time communication)

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Systems with Constraints

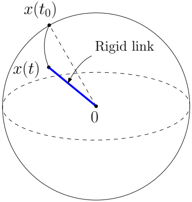
State-Space Constraints

Example

- Real constraint

$$\dot{x} = \pi^{\perp}(x)u$$

s.th. $\pi^{\perp}(x) := I - xx^{\top}$.



$\mathcal{X} = \mathbb{S}^n$

Systems with Constraints

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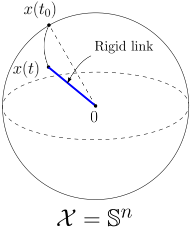
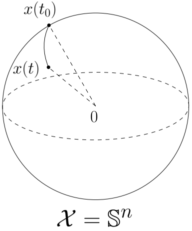


- Design constraint

$$\dot{x} = u$$
$$u = \pi^{\perp}(x)v$$



Consider the constrained system

$$\dot{x} = f(x, u) \qquad\qquad x \in \mathcal{X}$$

Objective (Control)

Design a control law $u(\cdot)$ such that $x = 0$ is a globally asymptotically (exponentially) stable equilibrium.

Systems with Constraints

Observer Design for Systems with State-Constraints

Consider the dynamical system

$$\dot{x} = f(x, u) \qquad\qquad x \in \mathcal{X}$$
$$y = h(x)$$

Objective (Estimation)

Design an estimation law for $\hat{x}$ such that $x - \hat{x} = 0$ a globally asymptotically (exponentially) stable equilibrium and

$$\hat{x} \in \hat{\mathcal{X}} \supseteq \mathcal{X}, \quad \textit{for all times.}$$

- $\hat{\mathcal{X}} = \mathcal{X}$: strictly constrained estimation
- $\hat{\mathcal{X}} \supset \mathcal{X}$: constrained estimation
- $\hat{\mathcal{X}} = \mathbb{R}^n$: non-constrained estimation

Outline

Topological Incompatibility

State-Space vs. Control Objective

Let $f \in \mathcal{C}^1(\mathbb{R}^n)$ and

$$\dot{x} = f(x), \quad x \in \mathcal{X} \tag{1}$$

[1] F. W. Wilson (1967). The structure of the level surfaces of a Lyapunov function. *Journal of Differential Equations*.

Topological Incompatibility

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Fact (Theorem 2.2 from [1])

The domain of asymptotic stability of **any critical point** *of* (1) *is* **diffeomorphic** *to* $\mathbb{R}^m$ *for some* $m \geq 0$*.*

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Corollary

If $\mathcal{X}$ and $\mathbb{R}^m$ are not topologically equivalent (not the same "shape") then global asymptotic stability is not possible.

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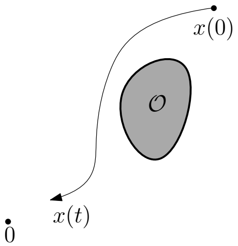
Topological Incompatibility

State-Space vs. Control Objective

Example (Non-simply connected $\mathcal{X}$)

- $\mathcal{X} = \mathbb{R}^n \setminus \mathcal{O}$ is **non-simply connected**



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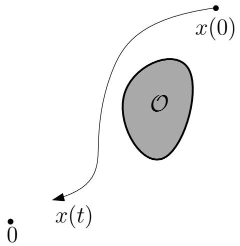
State-Space vs. Control Objective

Example (Non-simply connected $\mathcal{X}$)

- $\mathcal{X} = \mathbb{R}^n \setminus \mathcal{O}$ is **non-simply connected**
- $\mathbb{R}^m$ is **simply connected** $\forall m$



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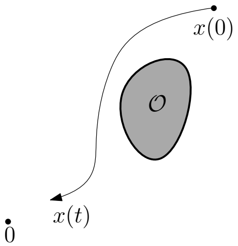


Corollary

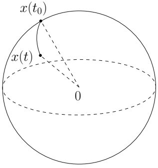
Topological obstruction to GAS on non-simply connected spaces.

Topological Incompatibility

State-Space vs. Control Objective

Example (Non-contractible $\mathcal{X}$)

- Compact manifolds are **non-contractible**



[2] $\mathcal{M}$ is contractible if there exists a continuous map $h : \mathcal{M} \times [0,1] \to \mathcal{M}$ and $x_0 \in \mathcal{M}$ such that $h(x,0) = x$ and $h(x,1) = x_0$ for all $x \in \mathcal{M}$.

Topological Incompatibility

State-Space vs. Control Objective

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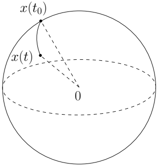
- $\mathbb{R}^n$ is **contractible**



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State-Space vs. Control Objective

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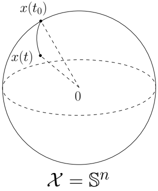

Corollary

Topological obstruction to GAS on non-contractible[2] spaces.

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Topological Incompatibility

State-Space vs. Control Objective

Example (Control on $\mathbb{S}^1$)

- Kinematics on $\mathbb{S}^1$

$$\begin{cases} \dot{x}_1 &= -\omega x_2 \\ \dot{x}_2 &= \omega x_1 \end{cases}$$

Topological Incompatibility

State-Space vs. Control Objective

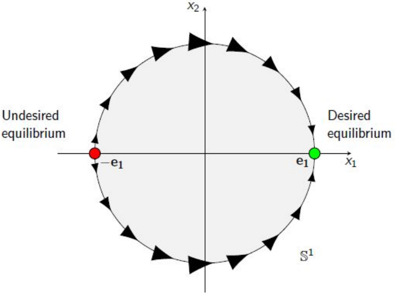
Example (Control on $\mathbb{S}^1$)

- Kinematics on $\mathbb{S}^1$

$$\begin{cases} \dot{x}_1 &= -\omega x_2 \\ \dot{x}_2 &= \omega x_1 \end{cases}$$

- Smooth Controller

$$\omega = -kx_2, \quad k > 0.$$



Example (Control on $\mathbb{S}^1$)

- Kinematics on $\mathbb{S}^1$

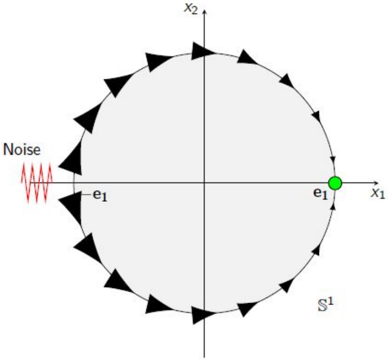
$$\begin{cases} \dot{x}_1 &= -\omega x_2 \\ \dot{x}_2 &= \omega x_1 \end{cases}$$

- Discontinuous Controller

$$\omega = -\mathrm{sign}(x_2)\,\arccos(x_1)$$

with

$$\mathrm{sign}(x_2) := \begin{cases} 1 & x_2 \geq 0 \\ -1 & x_2 < 0 \end{cases}$$



Topological Incompatibility

State-Space vs. Control Objective

Fact (C. Mayhew et al. 2011[3])

If a compact set can not be globally asymptotically stabilized by continuous feedback then it can not be **robustly asymptotically globally stabilized** *by discontinuous feedback either.*

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$\Rightarrow$ We need either:

- Hybrid Feedback
- Time-Varying Feedback

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$\Rightarrow$ We need either:

- **Hybrid Feedback**
- Time-Varying Feedback (open problem)

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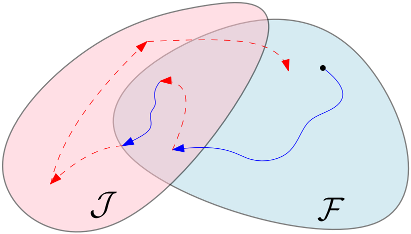
Outline

Hybrid Systems Framework

$$\begin{cases} \dot{x} & \in \mathbf{F}(x) \qquad x \in \mathcal{F} \\ x^+ & \in \mathbf{J}(x) \qquad x \in \mathcal{J} \end{cases}$$



- **state**: $x \in \mathbb{R}^n$
- **flow map**: $\mathbf{F} : \mathbb{R}^n \rightrightarrows \mathbb{R}^n$
- **flow set**: $\mathcal{F} \subseteq \mathbb{R}^n$
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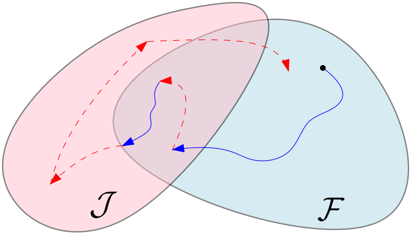

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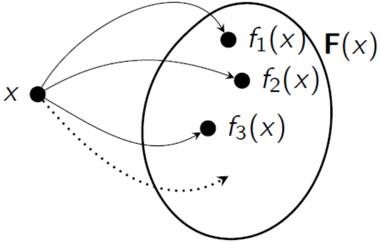

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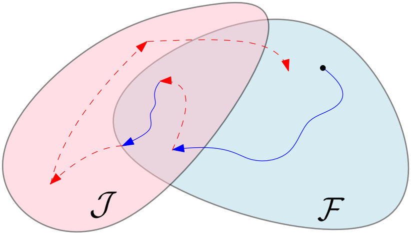

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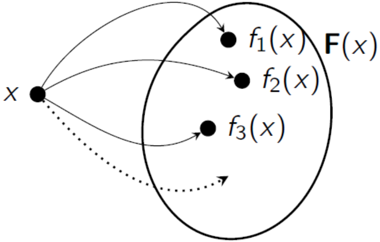

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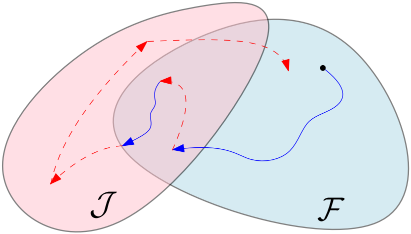

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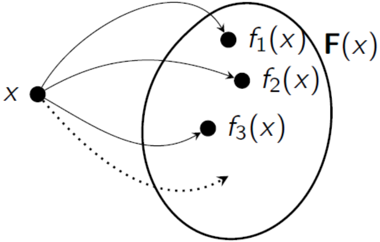

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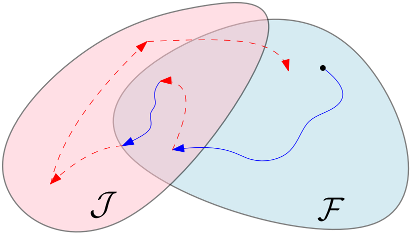

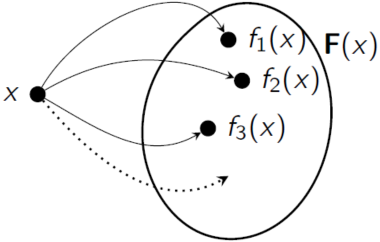

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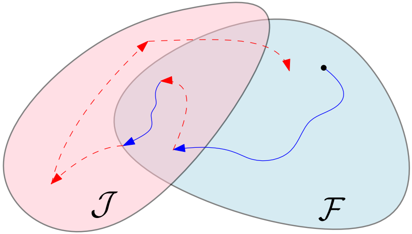

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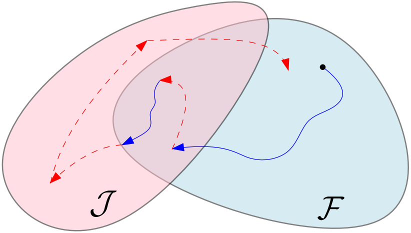

Hybrid time domain

$$(t, j) \in \mathbb{H} \subseteq \mathbb{R}_{\geq 0} \times \mathbb{N}$$

- t: amount of **time** passed
- j: number of **jumps** occurred

Hybrid Systems Framework

Hybrid Time Domains

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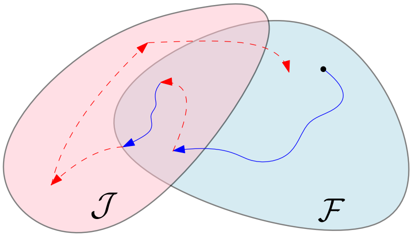

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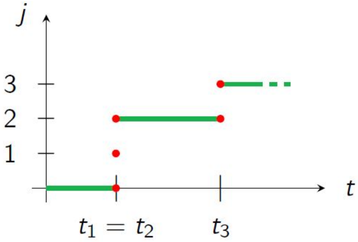

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Hybrid Systems Framework

Motivation Example on $\mathbb{S}^1$

- Kinematics on $\mathbb{S}^1$

$$\dot{x} = S(\omega)x$$

Hybrid Systems Framework

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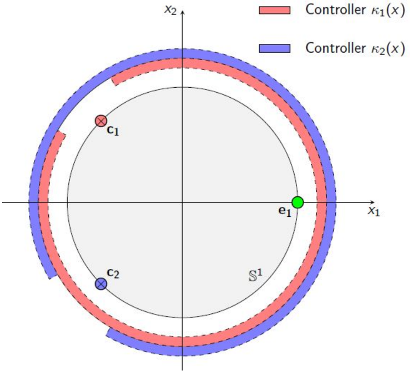
Let $q \in \{1, 2\}$ and $c_q \in \mathcal{C}_q \subset \mathbb{S}^1$.



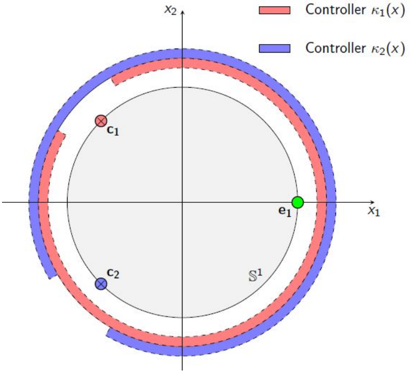
- Kinematics on $\mathbb{S}^1$

$$\dot{x} = S(\omega)x$$

Let $q \in \{1,2\}$ and $c_q \in \mathcal{C}_q \subset \mathbb{S}^1$.

$$\begin{cases} \dot{x} &= S(\kappa_q(x))x \\ \dot{q} &= 0 \end{cases} \quad x \in \overline{\mathbb{S}^1 \setminus \mathcal{C}_q}$$

$$\begin{cases} x^+ &= x \\ q^+ &= 3 - q \end{cases} \quad x \in \mathcal{C}_q$$



Outline

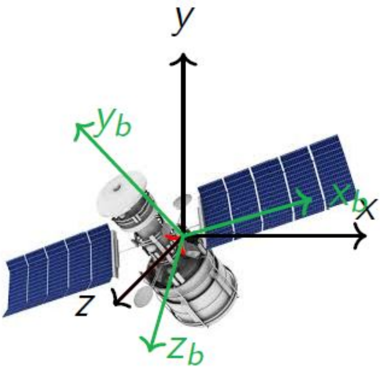
Hybrid Attitude Stabilization on $\mathbb{SO}(3)$

Problem Formulation

Consider the attitude kinematics

$$\dot{R} = R[\omega]_\times, \quad R(0) \in \mathbb{SO}(3)$$

where R is the attitude and $\omega \in \mathbb{R}^3$ is the angular velocity.



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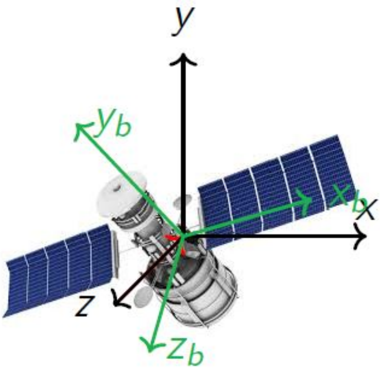
where R is the attitude and $\omega \in \mathbb{R}^3$ is the angular velocity.

Objective

Design ω such that $R = I_3$ is ***globally exponentially stable****.*

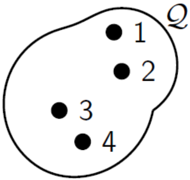


- $q \in \mathcal{Q}$ is a discrete variable (**controller mode**)

Definition

$\Phi(R, q)$ is a **potential function** if

- Continuous
- Continuously differentiable on $\mathcal{D} \subseteq \mathbb{SO}(3) \times \mathcal{Q}$
- Positive definite w.r.t $\{I\} \times \mathcal{Q}$



Hybrid Attitude Stabilization

Exp-synergistic Potential Functions

- $\mathcal{F} := \{(R, q) : \Phi(R, q) - \min_{m \in \mathcal{Q}} \Phi(R, m) \leq \delta\}$

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Definition

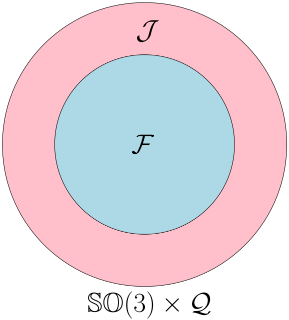
Φ is **exp-synergistic** with gap exceeding $\delta > 0$ iff:

$$\alpha_1 |R|_I^2 \leq \Phi(R, q) \leq \alpha_2 |R|_I^2 \qquad \forall (R, q) \in \mathbb{SO}(3) \times \mathcal{Q}$$
$$\alpha_3 |R|_I^2 \leq \|\nabla \Phi(R, q)\|_F^2 \leq \alpha_4 |R|_I^2 \qquad \forall (R, q) \in \mathcal{F}$$
$$\mathcal{F} \subseteq \mathcal{D}$$

- Both Φ and $\nabla\Phi$ are quadratic in $|R|_I := \|I - R\|_F / \sqrt{8}$
- Possible singular/critical points are outside the flow set $\mathcal{F}$

Hybrid Attitude Stabilization

Synergistic Hybrid Feedback

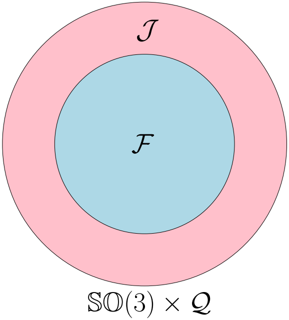


Hybrid Attitude Stabilization

Synergistic Hybrid Feedback

- Hybrid controller

$$\begin{aligned} \omega &= -(R^\top \nabla \Phi(R, q))^\vee \\ \dot{q} &= 0 && (R, q) \in \mathcal{F} \\ q^+ &\in \arg\min_{m \in \mathcal{Q}} \Phi(R, m) && (R, q) \in \mathcal{J} \end{aligned}$$



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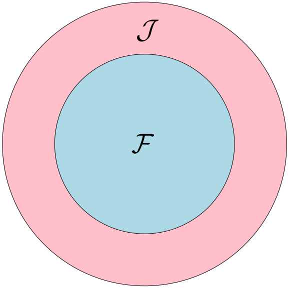
Theorem

If Φ is **exp-synergistic** *with gap exceeding δ then the set $\{I\} \times \mathcal{Q}$ is globally exponentially stable.*



Hybrid Attitude Stabilization

Construction of exp-synergistic potential functions

For example[4][5], consider

$$\Psi(R) := \mathbf{tr}(A(I - R)), \quad A = A^\top > 0.$$

Consider the map (angular warping2)

$$\Gamma(R, q) := R \exp(\theta(R, q)[u(q)]_\times)$$

[4] Berkane, S., Abdessameud, A., and Tayebi, A. (2017). Hybrid global exponential stabilization on $SO(3)$. Automatica, 81, 279-285.

[5] Berkane, S., Abdessameud, A., and Tayebi, A. (2018). Hybrid Output Feedback For Attitude Tracking on $SO(3)$. IEEE Transactions on Automatic Control.

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Theorem

Under some suitable $\theta(R, q)$ and $u(q)$, the potential function $\Psi \circ \Gamma$ is exp-synergistic with a given gap δ.

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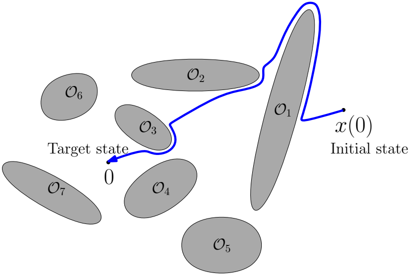
[5] Berkane, S., Abdessameud, A., and Tayebi, A. (2018). Hybrid Output Feedback For Attitude Tracking on $SO(3)$. IEEE Transactions on Automatic Control.

Obstacle Avoidance Problem

- Artificial potential fields suffer from local minima
- Navigation functions are not correct-by-construction (require tuning)
- Resulting feedback is strongly invasive



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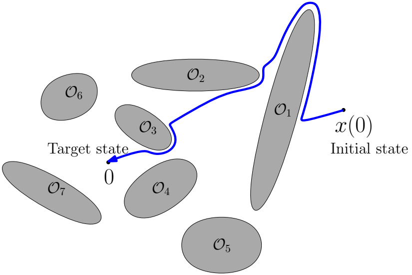

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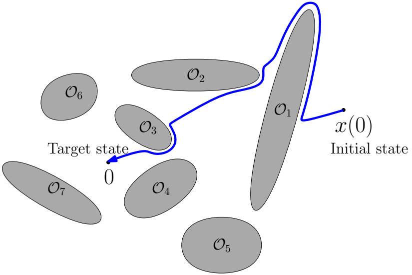

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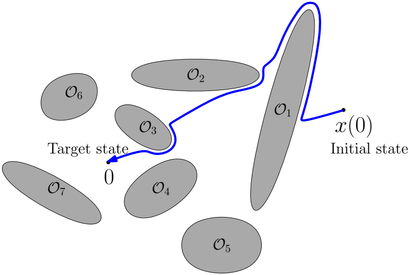
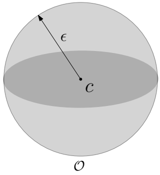


Consider the dynamics

$$\dot{x} = u, \quad x(0) \in \mathbb{R}^n \setminus \mathcal{O}$$

with the objective to globally asymptotically stabilize $x = 0$ while avoiding the obstacle

$$\mathcal{O} := \{x \in \mathbb{R}^n : \|x - c\| \leq \epsilon\}.$$



Obstacle Avoidance via Hybrid Feedback

Control Input

The control law has three modes

$$u = \kappa(x, m) := \begin{cases} -k_0 x, & m = 0 \\ -k_m \pi^{\perp}(x - c)(x - p_m), & m \in \{-1, 1\} \end{cases}$$

- Stabilization mode ($m = 0$)
- Avoidance modes ($m = -1, 1$) guarantee that

$$\frac{d\|x - c\|^2}{dt} = -k_m (x - c)^{\top} \pi^{\perp}(x - c)(x - p_m) = 0$$

$\Rightarrow$ safe avoidance

Obstacle Avoidance via Hybrid Feedback

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Under the feedback $-k_0 x$, what is the *unsafe* region?

Obstacle Avoidance via Hybrid Feedback

Construction of the Stabilization Flow and Jump Sets

Under the feedback $-k_0 x$, what is the *unsafe* region?

$$\frac{d\|x-c\|^2}{dt} = -2k_0 x^\top (x-c) \leq 0 \Longleftrightarrow \|x-c/2\|^2 \geq \|c/2\|^2 .$$

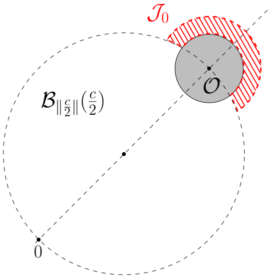
Obstacle Avoidance via Hybrid Feedback

Under the feedback $-k_0 x$, what is the *unsafe* region?

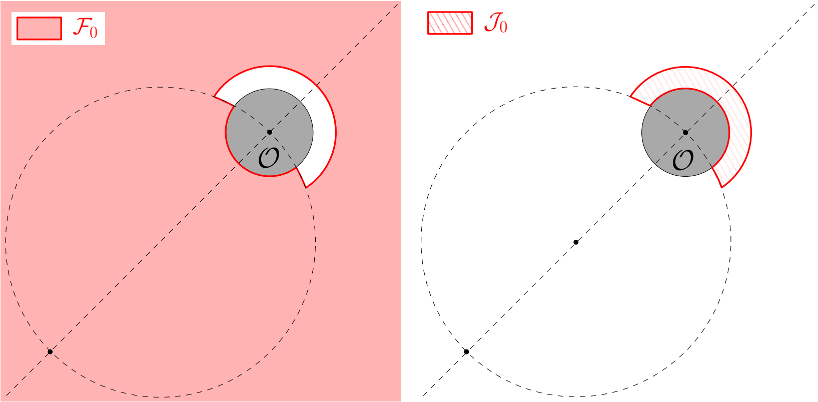
$$\frac{d\|x-c\|^2}{dt} = -2k_0 x^\top (x-c) \le 0 \iff \|x - c/2\|^2 \ge \|c/2\|^2 .$$

$\mathcal{J}_0$: safety helmet

We need to jump to the avoidance mode when $x \in \mathcal{J}_0$



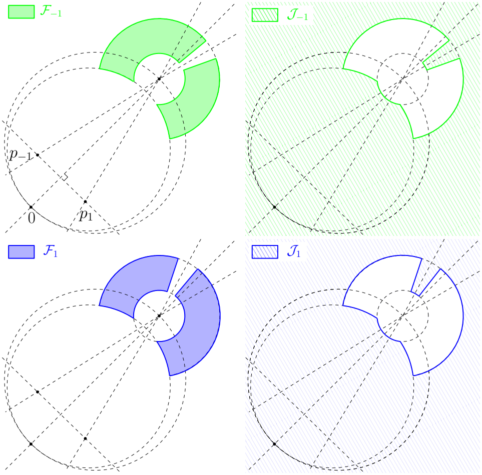
Stabilization flow set $\mathcal{F}_0$ and jump set $\mathcal{J}_0$



Obstacle Avoidance via Hybrid Feedback

Construction of the Avoidance Flow and Jump Sets

- p_1, p_{-1} are auxiliary attractive points
- The equilibria of the avoidance mode lie in the jump sets $\mathcal{J}_m, m = -1, 1$



Obstacle Avoidance via Hybrid Feedback

Closed-Loop Hybrid Dynamical System

$$\begin{cases} \dot{x} = \kappa(x, m) \\ \dot{m} = 0 \end{cases} \qquad (x, m) \in \bigcup_{m \in \{-1,0,1\}} \mathcal{F}_m \times \{m\} \tag{2}$$

$$\begin{cases} x^+ = x \\ m^+ \in \mathbf{M}(x, m) \end{cases} \qquad (x, m) \in \bigcup_{m \in \{-1,0,1\}} \mathcal{J}_m \times \{m\}. \tag{3}$$

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Theorem

- *(**Safety**)* $(\mathbb{R}^n \setminus \mathcal{O}) \times \{-1, 0, 1\}$ *is forward invariant*
- *(**Convergence**)* $\{0\} \times \{0\}$ *is globally asymptotically stable*
- *(**Preservation**) there exist controller parameters such that u matches $-k_0 x$ almost everywhere*

Obstacle Avoidance via Hybrid Feedback

Simulation (3D scenario)

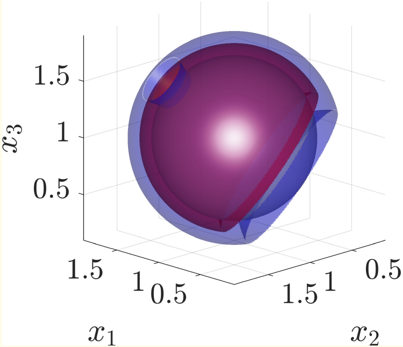

Figure 1: $\mathcal{F}_1$ (blue), $\mathcal{J}_0$ (red) and obstacle (gray)

Obstacle Avoidance via Hybrid Feedback

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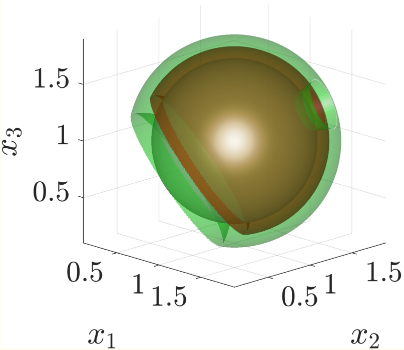

Figure 1: $\mathcal{F}_{-1}$ (green), $\mathcal{J}_0$ (red) and obstacle (gray)

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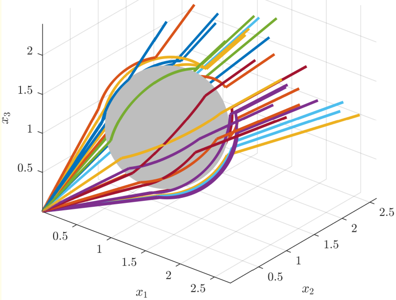
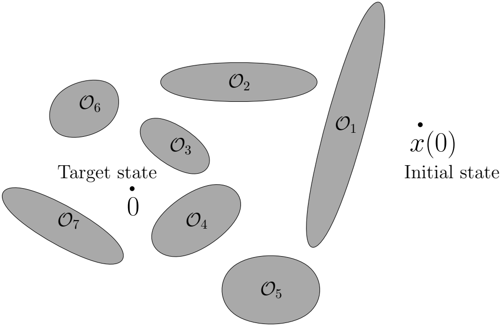


Figure 1: Different trajectories

Obstacle Avoidance via Hybrid Feedback

- Preliminary work for single spherical obstacle [6]
- We have extensions to **multiple ellipsoid-shaped** obstacles[7]



[6] S. Berkane, A. Bisoffi and D. V. Dimarogonas, "A Hybrid Controller for Obstacle Avoidance in the $n-$Dimensional Euclidean Space", submitted to *European Control Conference*, 2019.

[7] Journal extension in preparation.

Outline

Observer Design for Systems with State-Constraints

Objective (Estimation)

Design an estimation law for $\hat{x}$ such that $x - \hat{x} = 0$ is globally asymptotically stable and

$$\hat{x} \in \hat{\mathcal{X}} \supseteq \mathcal{X} \quad \textit{for all times.}$$

- $\hat{\mathcal{X}} = \mathcal{X}$: strictly constrained estimation
- $\hat{\mathcal{X}} \supset \mathcal{X}$: constrained estimation
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Hybrid Attitude Estimation on $\mathbb{SO}(3)$

Problem Formulation

Consider the attitude kinematics:

$$\dot{R} = R[\omega]_\times$$

with vector measurements on $\mathbb{S}^2$:

$$\underbrace{b_i}_{\text{measured}} = R^\top \underbrace{a_i}_{\text{known}}$$

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Objective

Estimate the **rotation matrix** *R using gyro readings of ω and vector measurements $b_1, \cdots, b_n$ with* **global exponential stability** *of the estimation error.*

- Attitude estimation

$$\dot{\hat{R}} = \hat{R}[\omega + \sigma]_\times, \quad \hat{R}(0) \in \mathbb{SO}(3)$$

- Strictly constrained since $\hat{R}(t) \in \mathbb{SO}(3)$ for all times.
- Cost function

$$\frac{1}{2}\sum_{i=1}^{n} \|b_i - \hat{R}^\top a_i\|^2 \overset{\text{gradient}}{\rightsquigarrow} \sigma = \sum_{i=1}^{n} \rho_i (b_i \times \hat{R}^\top a_i)$$

Theorem (R. Mahony & T. Hamel)

Almost *global asymptotic stability and* **local** *exponential stability.*

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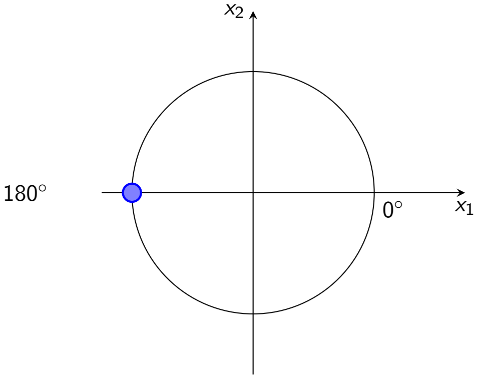
Motivation on $\mathbb{S}^1$



Hybrid Attitude Estimation on $\mathbb{SO}(3)$

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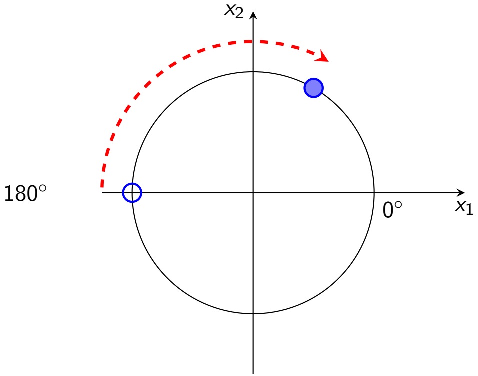
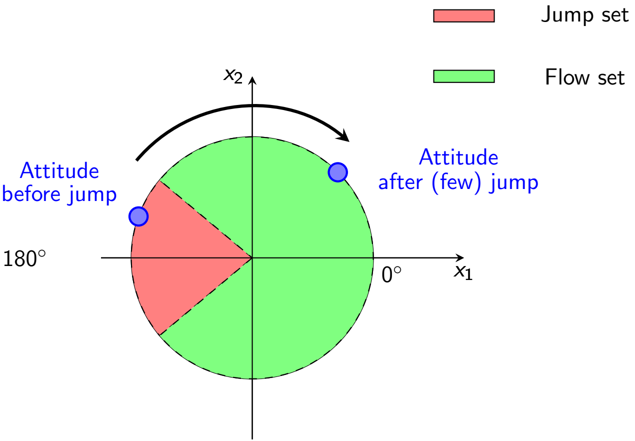


- State **reset**: $\hat{x}^+ = \mathbf{R}(\theta)\hat{x}$
- Causes estimation error to decrease

Hybrid Attitude Estimation on $\mathbb{SO}(3)$

Motivation on $\mathbb{S}^1$



Hybrid Attitude Estimation on $\mathbb{SO}(3)$

Reset-based approach

- Attitude estimation

$$\dot{\hat{R}} = \hat{R}[\omega + \sigma]_\times \qquad ((b_i)_{1\cdots n}, \hat{R}) \in \hat{\mathcal{F}}$$
$$\hat{R}^+ = \mathbf{R}(\theta u_q)\hat{R} \qquad ((b_i)_{1\cdots n}, \hat{R}) \in \hat{\mathcal{J}}$$

- Innovation

$$\textbf{(Cost function)} \quad \hat{\Phi}((b_i)_{1\cdots n}, \hat{R}) \rightsquigarrow^{gradient} \sigma = \hat{\mathbf{w}}((b_i)_{1\cdots n}, \hat{R})$$

- Flow and jump sets

$$\hat{\mathcal{F}} = \{\hat{\Phi}(\cdot, \hat{R}) - \min_m \hat{\Phi}(\cdot, \mathbf{R}(\theta u_q)\hat{R}) \leq \delta\}$$
$$\hat{\mathcal{J}} = \{\hat{\Phi}(\cdot, \hat{R}) - \min_m \hat{\Phi}(\cdot, \mathbf{R}(\theta u_q)\hat{R}) \geq \delta\}$$

- $\theta \in \mathbb{R}$ and $q = \arg\min_m \hat{\Phi}(\cdot, \mathbf{R}(\theta u_q)\hat{R})$.

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Hybrid Attitude Estimation on $\mathbb{SO}(3)$

Reset-based approach

Theorem (S. Berkane, PhD thesis 2017)

If we have

1. $\hat{\Phi}$ *is quadratic*
2. $\|\nabla\hat{\Phi}\|_F^2$ *is quadratic on the flow set* $\hat{\mathcal{F}}$
3. *singular points of* $\hat{\Phi}$ *lie in the jump set* $\hat{\mathcal{J}}$

then the zero estimation error is **globally exponentially stable**.

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2. $\|\nabla\hat{\Phi}\|_F^2$ *is quadratic on the flow set* $\hat{\mathcal{F}}$
3. *singular points of* $\hat{\Phi}$ *lie in the jump set* $\hat{\mathcal{J}}$

then the zero estimation error is **globally exponentially stable**.

Example

- Take $\Phi = \sum_{i=1}^n \rho_i \|b_i - \hat{R}^\top a_i\|^2$
- Pick $\theta = \pi$ and $0 < \delta < \lambda_1^A + \lambda_2^A$ with $A = \sum_{i=1}^n \rho_i a_i a_i^\top$
- Pick $\{u_1, u_2, u_3\}$ be an orthonormal set of eigenvectors of A

Hybrid Attitude Estimation on $\mathbb{SO}(3)$

- **Attitude + gyro bias**[8]: synergistic-based approach
- **GPS-aided**[9]: reset-based approach
- **Full state**[10]: attitude and angular velocity estimation, synergistic-based approach
- **Intermittent measurements**[11]: sensors with different sampling

[8]Berkane, S., Abdessameud, A., & Tayebi, A. (2017). Hybrid Attitude and Gyro-Bias Observer Design on $SO(3)$. IEEE TAC.

[9]Berkane, S. and Tayebi, A. (2017). Attitude and gyro bias estimation using GPS and IMU measurements. IEEE CDC.

[10]Berkane, S., Abdessameud, A., & Tayebi, A. (2018). Hybrid Output Feedback For Attitude Tracking on SO (3). IEEE TAC.

[11]Berkane, S. and Tayebi, A. (2018). Attitude Estimation with Intermittent Measurements. Automatica.

Observer Design for Systems with State-Constraints

Objective (Estimation)

Design an estimation law for $\hat{x}$ *such that* $x - \hat{x} = 0$ *is globally asymptotically stable and*

$$\hat{x} \in \hat{\mathcal{X}} \supseteq \mathcal{X} \quad \textit{for all times.}$$

- $\hat{\mathcal{X}} = \mathcal{X}$: strictly constrained estimation
- $\hat{\mathcal{X}} \supset \mathcal{X}$: constrained estimation
- $\hat{\mathcal{X}} = \mathbb{R}^n$: non-constrained estimation

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Consider the LTV system

$$\dot{x} = A(t)x + B(t)u(t)$$
$$y = C(t)x$$

Hybrid Constrained Estimation For LTV Systems

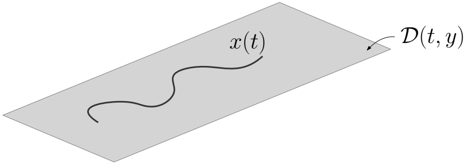
System Model

Consider the LTV system

$$\dot{x} = A(t)x + B(t)u(t)$$
$$y = C(t)x$$

- x is **constrained** to evolve on the set

$$\mathcal{D}(t, y) := \{x \in \mathbb{R}^n : D(t, y)x = d(t, y)\}$$

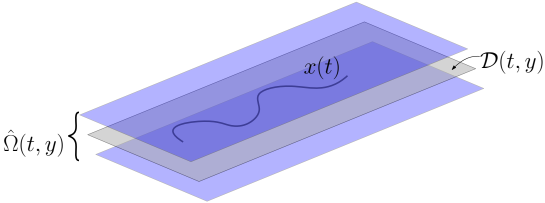


Hybrid Constrained Estimation For LTV Systems

Estimation Objective

Objective

Design an estimator for $\hat{x}$ s.th. $\hat{x} \in \hat{\Omega}(t, y) \supset \mathcal{D}(t, y)$ for all times.



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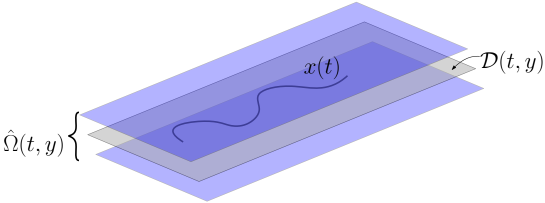

- (Part of) the **output equation** $C(t)x = y$ can be used as a constraint in $D(t, y)x = d(t, y)$.

We propose the following hybrid observer:

$$\begin{aligned} \dot{\hat{x}} &= A(t)\hat{x} + B(t)u(t) + K(t)(y - C(t)\hat{x}), && \hat{x} \in \hat{\mathcal{F}}(t, y) \\ \hat{x}^+ &= \mathbf{P}_{\mathcal{D}(t,y)}(\hat{x}), && \hat{x} \in \hat{\mathcal{J}}(t, y) \end{aligned}$$

- $K(t)$: from a Riccati equation.
- $\mathbf{P}_{\mathcal{D}}(\hat{x})$: projection operator on $D\hat{x} = d$ given by

$$\mathbf{P}_{\mathcal{D}}(\hat{x}) := \hat{x} - PD^\top(DPD^\top)^{-1}(D\hat{x} - d)$$

for some positive definite matrix P.

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Hybrid Constrained Estimation For LTV Systems

Design of the Flow and Jump Sets

- (**Objective**): $\hat{\mathcal{F}}(t, y) \subset \hat{\Omega}(t, y)$
- (**Complete**): $\mathrm{dist}(\hat{\mathcal{J}}(t, y), \hat{\mathcal{D}}(t, y)) > 0$
- (**Global**): $\hat{\mathcal{F}}(t, y) \cup \hat{\mathcal{J}}(t, y) = \mathbb{R}^n$

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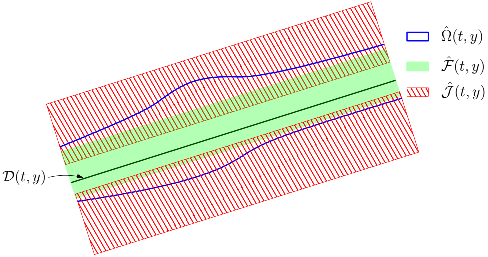
Example



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Theorem

Global exponential stability of the zero estimation error.

Simulation (A vehicle moving on a road with angle θ)

Let

$$\dot{x} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I \end{bmatrix} u(t)$$
$$y = \begin{bmatrix} I & 0 \end{bmatrix} x$$

with

$$u(t) = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} v(t)$$

which leads to the constraint

$$\begin{bmatrix} \sin(\theta) & -\cos(\theta) & 0 & 0 \end{bmatrix}^\top x(t) = 0.$$

Simulation (A vehicle moving on a road with angle θ)

- Objective set

$$\hat{\Omega} = \{\hat{x} \in \mathbb{R}^4 : |\sin(\theta)\hat{x}_1 - \cos(\theta)\hat{x}_2| \leq \mu\}.$$

- Flow set

$$\hat{\mathcal{F}} = \{\hat{x} \in \mathbb{R}^4 : |\sin(\theta)\hat{x}_1 - \cos(\theta)\hat{x}_2| \leq \epsilon\}.$$

- Jump set

$$\hat{\mathcal{J}} = \{\hat{x} \in \mathbb{R}^4 : |\sin(\theta)\hat{x}_1 - \cos(\theta)\hat{x}_2| \geq \epsilon\}.$$

with $0 < \epsilon < \mu$.

Hybrid Constrained Estimation For LTV Systems

Simulation Scenario

Simulation (A vehicle moving on a road with angle θ)

vidvid

Conclusion

- Systems with constraints present topological difficulties
- Hybrid systems tools are promising for the control & estimation of systems with constraints
- Future work needed for complex and multirobotic systems

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