Nonlinear Estimation for Position-Aided Inertial Navigation Systems

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Problem Formulation

Kinematics

Kinematics of a rigid-body vehicle:

$$\begin{cases} \dot{p} &= v, \\ \dot{v} &= g + a_I, \\ \dot{R} &= R[\omega]_\times, \end{cases}$$

- $p \in \mathbb{R}^3$: inertial position of the vehicle's center of gravity
- $v \in \mathbb{R}^3$: inertial linear velocity
- $R \in \mathbb{SO}(3)$: rotation matrix of $\{B\}$ with respect to $\{I\}$
- $\omega \in \mathbb{R}^3$: angular velocity expressed in $\{B\}$
- g: acceleration due to gravity
- a_I: "apparent acceleration", capturing all non-gravitational forces applied to the vehicle, expressed in $\{I\}$.

Problem Formulation

Inertial sensors

Inertial measurement unit (IMU):

$$\omega^{y} = \omega + b_{\omega}, \quad (1)$$

$$a_B = R^{\top} a_I, \quad (2)$$

$$m_B = R^{\top} m_I, \quad (3)$$

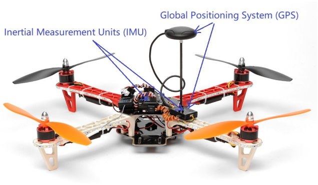
- b_{ω}: a constant unknown gyro bias
- m_I: a constant and know earth's magnetic field

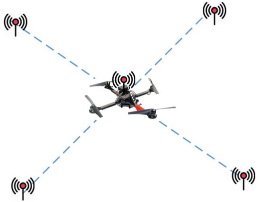
Assumption

There exists a constant $c_0 > 0$ such that $\|m_I \times a_I(t)\| \geq c_0$ for all $t \geq 0$.

Problem Formulation

Position measurements





Position output vector:

$$y = C_p p, \tag{4}$$

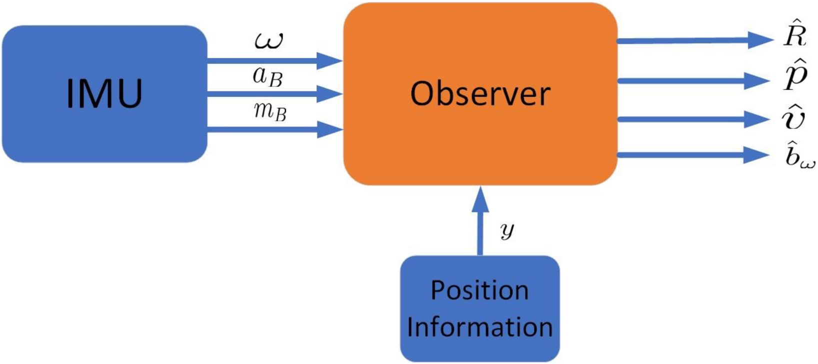
- C_p: constant and known $(m \times 3)$ output matrix

Assumption

$\mathrm{rank}(C_p) = 3.$

Problem Formulation

Objective



Translational Dynamics

Let $x := (p, v) \in \mathbb{R}^6$. The dynamics of x are written as:

$$\dot{x} = Ax + B(g + a_I), \quad (5)$$

$$y = Cx, \quad (6)$$

where the matrices A, B and C are defined as follows:

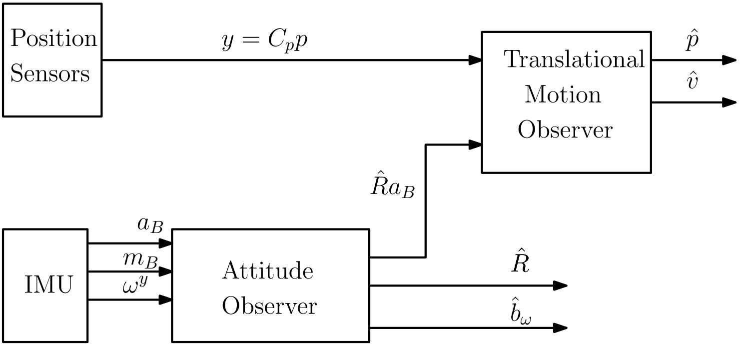
$$A = \begin{bmatrix} 0_{3\times3} & I_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}, B := \begin{bmatrix} 0_{3\times3} \\ I_3 \end{bmatrix}, C = \begin{bmatrix} C_p^\top \\ 0_{3\times m} \end{bmatrix}^\top. \quad (7)$$

- Linear time-invariant system with unknown input a_I
- We only measure $a_B = R^\top a_I$ (in body-frame)

Common estimation approach (ad-hoc method)

Assumption

Negligible acceleration, i.e., $a_I \approx -g$.



Common estimation approach (ad-hoc method[1])

Assumption

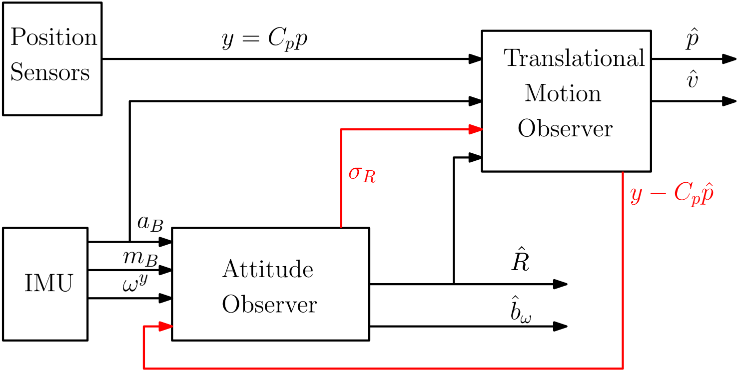
Negligible acceleration, i.e., $a_I \approx -g$.

$$\text{Attitude observer} \begin{cases} \dot{\hat{R}} &= \hat{R}[\omega_y - \hat{b}_\omega + k_R\sigma_R]_\times, \\ \dot{\hat{b}}_\omega &= Proj(\hat{b}_\omega, -k_b\sigma_R), \\ \sigma_R &= \rho_1(m_B \times \hat{R}^\top m_I) + \rho_2(a_B \times \hat{R}^\top(-g)). \end{cases}$$

$$\text{Translational Observer} \begin{cases} \dot{\hat{x}} = A\hat{x} + B(g + \hat{R}a_B) + K(y - C\hat{x}) \\ (A - KC) \text{ is Hurwitz} \end{cases}$$

[1] R. Mahony, T. Hamel and J. Pflimlin, "**Nonlinear Complementary Filters on the Special Orthogonal Group**," in IEEE TAC, 2008.
H. F. Grip *et al.* "**Attitude Estimation Using Biased Gyro and Vector Measurements With Time-Varying Reference Vectors**," in IEEE TAC, 2012.

Proposed estimation approach



- The proposed observer introduces coupling (in red) between the translational estimator and the rotational estimator through their innovation terms.
- This additional coupling is important to guarantee the stability of the observer **without the "small" acceleration assumption**.

Translational Motion Observer

Let $K = L_\gamma K_0$ with $L_\gamma = \mathrm{blockdiag}(\gamma I_3, \gamma^2 I_3)$ and $\gamma \geq 1$:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + B(ge_3 + \hat{R}a_B) + K(y - C\hat{x}) + \sigma_x, \\ \sigma_x = -k_R(A - KC)^{-1}B[\hat{R}\sigma_R]_\times \hat{R}a_B, \\ (A - K_0C) \text{ is Hurwitz} \end{cases}$$

- If $\hat{R} \to R$, this is equivalent to the **Luenberger-type observer**.
- The matrix L_γ is introduced to assign a certain time-scaling structure between the different estimation errors.

Rotational Motion Observer

Consider the "nonlinear complementary filter"-type observer[2]

$$\begin{cases} \dot{\hat{R}} & = \hat{R}[\omega^y - \hat{b}_\omega + k_R\sigma_R]_\times \\ \dot{\hat{b}}_\omega & = \text{Proj}(\hat{b}_\omega, -k_R\sigma_R) \end{cases}$$

$$\sigma_R = \rho_1(m_B \times \hat{R}^\top m_I) + \rho_2(a_B \times \hat{R}^\top \textbf{sat}(B^\top K(y - C\hat{x}))).$$

- Proj: is the parameter projection function
- sat: is a saturation function

[2] R. Mahony, T. Hamel and J. Pflimlin, "**Nonlinear Complementary Filters on the Special Orthogonal Group**," in IEEE TAC, 2008.
H. F. Grip, T. I. Fossen, T. A. Johansen and A. Saberi, "**Attitude Estimation Using Biased Gyro and Vector Measurements With Time-Varying Reference Vectors**," in IEEE TAC, 2012.

Main Result

Assumption

- *m_I and $a_I(t)$ are not collinear for all times*
- *$\omega, b_\omega, a_I, \dot{a}_I$ are uniformly bounded*

Theorem (Semi-global exponential stability)

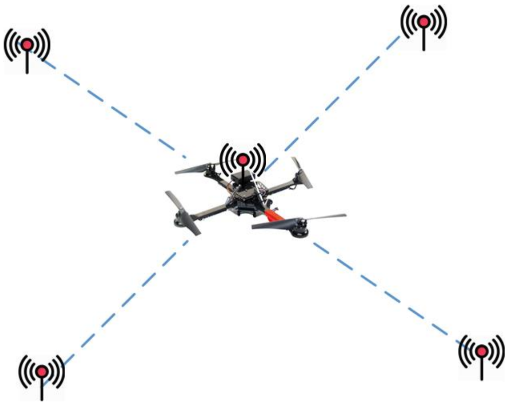
For all initial conditions (except attitude errors at 180°), there exist (high) gains such that the estimation error converges exponentially to zero.

- Although, sufficiently large gains are needed in the proof, simulation results demonstrate that this is very conservative.
- To prove the convergence of the estimation errors, we introduce the following auxiliary error variable

$$\zeta := L_\gamma^{-1}\left[(A - KC)\tilde{x} + B(I - \tilde{R})^\top a_I\right]. \qquad (8)$$

Simulation Results

Range measurements (e.g., Ultra-wide-Band (UWB))



Simulation Results

Range measurements (e.g., Ultra-wide-Band (UWB))

At least 4 non-coplanar source points are needed located at a_i.

$$d_i = \|p - a_i\|, \quad i = 1, \cdots, n.$$

Output equation:

$$y_i := \frac{1}{2}\left(d_i^2 - d_1^2 - \|a_i\|^2 + \|a_1\|^2\right), \quad i = 2, \cdots, n.$$

$$y = \begin{bmatrix} (p_1 - a_2)^\top \\ \vdots \\ (p_1 - a_n)^\top \end{bmatrix} p := C_p p \tag{9}$$

Simulation Results

Range measurements (e.g., Ultra-wide-Band (UWB))

- Circular trajectory (with acceleration):

$$p(t) = \begin{bmatrix} \cos(2\pi t^2/100) \\ \sin(2\pi t^2/100) \\ 1 \end{bmatrix}. \tag{10}$$

- Angular velocity:

$$\omega(t) = \begin{bmatrix} \sin(0.2t) \\ \cos(0.1t) \\ \sin(0.3t + \pi/6) \end{bmatrix}, \tag{11}$$

- Anchors:

$$a_1 = [1\ 1\ 2]^\top, \tag{12}$$

$$a_2 = [1\ 3\ \ 0]^\top, \tag{13}$$

$$a_3 = [0\ 1\ 1]^\top, \tag{14}$$

$$a_4 = [6\ 5\ 5]^\top. \tag{15}$$

Simulation Results

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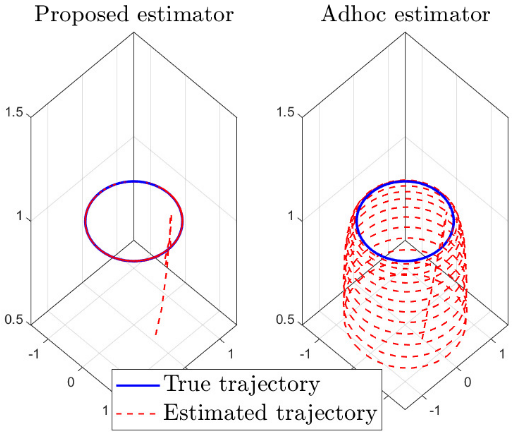

Figure 1: As the acceleration increases, the adhoc estimator drifts away from the true trajectory while the proposed estimator is stable. URL: `https://youtu.be/zbkSDZgh3vU`

Simulation Results

Range measurements (e.g., Ultra-wide-Band (UWB))

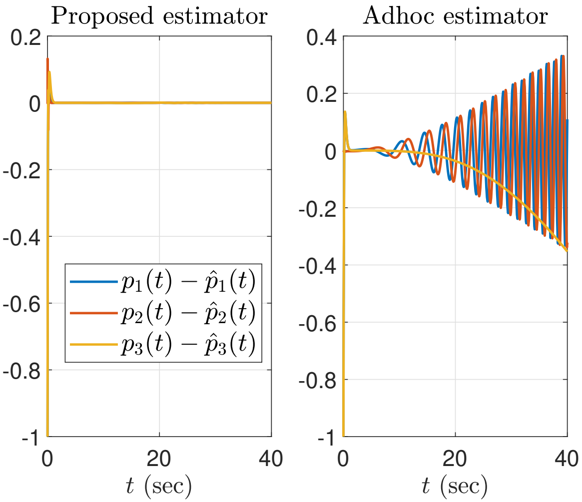

Figure 2: Position estimation errors.

Conclusion

- A nonlinear navigation observer with **semi-global exponential stability**
- Suitable in applications with **non-negligible linear accelerations** where the traditional cascaded approach fails
- Does not require the introduction of auxiliary states compared to previous works

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Thank you

Questions?