Hybrid Constrained Estimation For Linear Time-Varying Systems

Soulaimane Berkane [1] Abdelhamid Tayebi [2] Andrew Teel [3]

[1]KTH Royal Institute of Technology

[2]Lakehead University

[3]University of California, Santa Barbara

December 19, 2018

Outline

Motivation

Why do we need constrained estimation?

- Interconnection observer-controller where the controller is not well defined in certain regions
- Meaningful state estimates
  - Mass is positive
  - In chemical processes concentrations and pressures are nonnegative
  - Quaternion representation (unit norm)
- Constraints provide additional information which can be used in the estimator to speed up the convergence

System Model

Linear Time-Varying System

Consider the LTV system

$$\dot{x} = A(t)x + B(t)u(t)$$
$$y = C(t)x$$

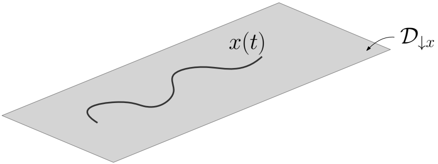
- $A(t), B(t), C(t)$ are $\mathcal{C}^1$ functions, uniformly bounded with bounded derivatives.
- The input $u(\cdot)$ is locally integrable.
- The pair $(A(\cdot), C(\cdot))$ is uniformly observable.

Constraint Set

Linear Equality Constraints

- Assume that (t, y, x) is **constrained to evolve** on the set

$$\mathcal{D} := \{(t, y, x) : D(t, y)x = d(t, y)\}$$



Assumption

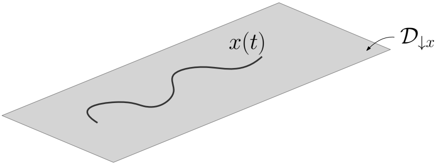
$D(t, y)$ is full raw rank and $D(t, y), d(t, y)$ are known, for all times

Constraint Set

Linear Equality Constraints

- Assume that (t, y, x) is **constrained to evolve** on the set

$$\mathcal{D} := \{(t, y, x) : D(t, y)x = d(t, y)\}$$



Assumption

$D(t, y)$ is full raw rank and $D(t, y), d(t, y)$ are known, for all times

- All or some rows of the **output equation** $C(t)x = y$ can be used in the constraint $D(t, y)x = d(t, y)$.

Kalman-Bucy Estimator

- Consider the Luenberger-type observer

$$\dot{\hat{x}} = A(t)\hat{x} + B(t)u(t) + K(t)(y - C(t)\hat{x})$$

with $K(t) = P(t)C(t)^\top Q(t)$ and

$$\dot{P} = AP + PA^\top - PC^\top QCP + V$$

- We can show UGES
- However, there is no guarantee that $\hat{x}$ remains in a given **region of interest**.

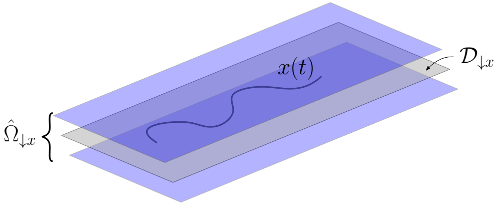
Constrained Estimation Objective

Objective

Design a hybrid observer for $\hat{x}$ such that

$$(t, y, \hat{x}) \in \hat{\Omega} \quad \textit{for all times},$$

where $\hat{\Omega} \supset \mathcal{D}$ is a given region of interest.



Projection on Linear Surfaces

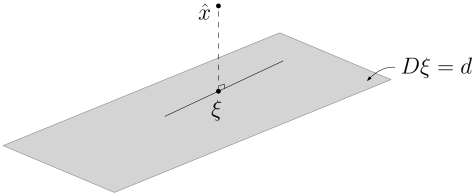
- Weighed projection on the surface $D\hat{x} = d$

$$\min_{\xi \in \mathbb{R}^n} (\xi - \hat{x})^\top P^{-1} (\xi - \hat{x}) \quad \text{s.t. } D\xi = d$$

- Solution is given explicitly by

$$\xi^* = \hat{x} - PD^\top (DPD^\top)^{-1} (D\hat{x} - d).$$

- Example with $P = I$ (orthogonal projection)



Proposed Hybrid Observer

We propose the following hybrid observer:

$$\dot{\hat{x}} = A(t)\hat{x} + B(t)u(t) + K(t)(y - C(t)\hat{x}), \quad (t, y, \hat{x}) \in \hat{\mathcal{F}},$$
$$\hat{x}^+ = \Pi_{P(t)}(\hat{x}, D(t, y), d(t, y)), \quad (t, y, \hat{x}) \in \hat{\mathcal{J}},$$

- $K(t) = P(t)C(t)^\top Q(t)$: optimal gain
- $P(t)$: solution of the Riccati equation
- $\Pi_P(\hat{x}, D, d)$: weighted projection operator on $D\hat{x} = d$

$$\Pi_P(\hat{x}, D, d) := \hat{x} - PD^\top(DPD^\top)^{-1}(D\hat{x} - d).$$

Proposed Hybrid Observer

We propose the following hybrid observer:

$$\begin{aligned} \dot{\hat{x}} &= A(t)\hat{x} + B(t)u(t) + K(t)(y - C(t)\hat{x}), \quad && (t, y, \hat{x}) \in \hat{\mathcal{F}}, \\ \hat{x}^{+} &= \Pi_{P(t)}(\hat{x}, D(t, y), d(t, y)), && (t, y, \hat{x}) \in \hat{\mathcal{J}}, \end{aligned}$$

- $K(t) = P(t)C(t)^{\top}Q(t)$: optimal gain
- $P(t)$: solution of the Riccati equation
- $\Pi_P(\hat{x}, D, d)$: weighted projection operator on $D\hat{x} = d$

$$\Pi_P(\hat{x}, D, d) := \hat{x} - PD^{\top}(DPD^{\top})^{-1}(D\hat{x} - d).$$

Proposed Hybrid Observer

We propose the following hybrid observer:

$$\begin{aligned} \dot{\hat{x}} &= A(t)\hat{x} + B(t)u(t) + K(t)(y - C(t)\hat{x}), \quad && (t, y, \hat{x}) \in \hat{\mathcal{F}}, \\ \hat{x}^{+} &= \Pi_{P(t)}(\hat{x}, D(t,y), d(t,y)), && (t, y, \hat{x}) \in \hat{\mathcal{J}}, \end{aligned}$$

- $K(t) = P(t)C(t)^{\top}Q(t)$: optimal gain
- $P(t)$: solution of the Riccati equation
- $\Pi_P(\hat{x}, D, d)$: weighted projection operator on $D\hat{x} = d$

$$\Pi_P(\hat{x}, D, d) := \hat{x} - PD^{\top}(DPD^{\top})^{-1}(D\hat{x} - d).$$

Proposed Hybrid Observer

We propose the following hybrid observer:

$$\begin{aligned} \dot{\hat{x}} &= A(t)\hat{x} + B(t)u(t) + K(t)(y - C(t)\hat{x}), && (t, y, \hat{x}) \in \hat{\mathcal{F}}, \\ \hat{x}^+ &= \Pi_{P(t)}(\hat{x}, D(t, y), d(t, y)), && (t, y, \hat{x}) \in \hat{\mathcal{J}}, \end{aligned}$$

- $K(t) = P(t)C(t)^\top Q(t)$: optimal gain
- $P(t)$: solution of the Riccati equation
- $\Pi_P(\hat{x}, D, d)$: weighted projection operator on $D\hat{x} = d$

$$\Pi_P(\hat{x}, D, d) := \hat{x} - PD^\top (DPD^\top)^{-1}(D\hat{x} - d).$$

Design of the Flow and Jump Sets

- The flow set is a subset of the target set

$$\hat{\mathcal{F}} \subset \hat{\Omega}$$

- **Strict decrease** of the error during the jumps as long as

$$\mathrm{dist}(\hat{\mathcal{J}}, \hat{\mathcal{D}}) > 0$$

- **Global** observer

$$\hat{\mathcal{F}} \cup \hat{\mathcal{J}} = \mathbb{R}_{\geq 0} \times \mathbb{R}^p \times \mathbb{R}^n$$

Design of the Flow and Jump Sets

- The flow set is a subset of the target set

$$\hat{\mathcal{F}} \subset \hat{\Omega}$$

- **Strict decrease** of the error during the jumps as long as

$$\mathrm{dist}(\hat{\mathcal{J}}, \hat{\mathcal{D}}) > 0$$

- **Global** observer

$$\hat{\mathcal{F}} \cup \hat{\mathcal{J}} = \mathbb{R}_{\geq 0} \times \mathbb{R}^p \times \mathbb{R}^n$$

Design of the Flow and Jump Sets

- The flow set is a subset of the target set

$$\hat{\mathcal{F}} \subset \hat{\Omega}$$

- **Strict decrease** of the error during the jumps as long as

$$\mathrm{dist}(\hat{\mathcal{J}}, \hat{\mathcal{D}}) > 0$$

- **Global** observer

$$\hat{\mathcal{F}} \cup \hat{\mathcal{J}} = \mathbb{R}_{\geq 0} \times \mathbb{R}^p \times \mathbb{R}^n$$

Design of the Flow and Jump Sets

- The flow set is a subset of the target set

$$\hat{\mathcal{F}} \subset \hat{\Omega}$$

- **Strict decrease** of the error during the jumps as long as

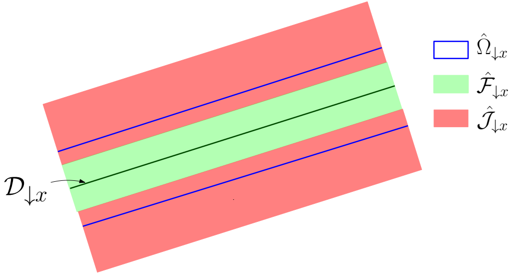
$$\mathrm{dist}(\hat{\mathcal{J}}, \hat{\mathcal{D}}) > 0$$

- **Global** observer

$$\hat{\mathcal{F}} \cup \hat{\mathcal{J}} = \mathbb{R}_{\geq 0} \times \mathbb{R}^p \times \mathbb{R}^n$$

Design of the Flow and Jump Sets

Example



Main Result

Theorem

The set $\mathcal{A} = \{(t, x, \hat{x}) : x = \hat{x}\}$ is uniformly globally asymptotically stable and globally exponentially stable in the $t-$direction. Moreover, $(t, y(t), \hat{x}_{\downarrow}(t)) \in \hat{\Omega}$ for all times.

Proof.

- Lyapunov function $\mathbf{V}(t, x, \hat{x}) = (x - \hat{x})^{\top} P(t)^{-1}(x - \hat{x})$
- During the flows, $\dot{\mathbf{V}} \leq -\lambda \mathbf{V}$
- During the jumps, $\mathbf{V}^{+} - \mathbf{V} \leq -\epsilon$, as long as $\mathrm{dist}(\hat{\mathcal{J}}, \hat{\mathcal{D}}) > 0$
- Number of jumps is finite and every solution is complete
- Pre-asymptotic stability and completeness $\rightarrow$ uniform global asymptotic stability.
- $|X(t, j)|_{\mathcal{A}} \leq k \exp(-\lambda t)|X(0, 0)|_{\mathcal{A}}$.
- $|X(t, j)|_{\mathcal{A}} \leq \kappa(|X(0, 0)|_{\mathcal{A}}) \exp(-\lambda(t + j))|X(0, 0)|_{\mathcal{A}}$.

Simulation Scenario

Simulation (A vehicle moving on a road with angle θ)

Let $x \in \mathbb{R}^4$ such that (x_1, x_2) is the (measured) position and (x_3, x_4) is the velocity. Second order dynamics:

$$\dot{x} = \begin{bmatrix} 0 & I_2 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ I_2 \end{bmatrix} u(t)$$
$$y = \begin{bmatrix} I_2 & 0 \end{bmatrix} x$$

with

$$u(t) = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} v(t)$$

which leads to the constraint

$$\begin{bmatrix} \sin(\theta) & -\cos(\theta) & 0 & 0 \end{bmatrix}^\top x(t) = 0.$$

Simulation Scenario

Simulation (A vehicle moving on a road with angle θ)

- Objective set

$$\hat{\Omega} = \{\hat{x} \in \mathbb{R}^4 : |\sin(\theta)\hat{x}_1 - \cos(\theta)\hat{x}_2| \leq \mu\}.$$

- Flow set

$$\hat{\mathcal{F}} = \{\hat{x} \in \mathbb{R}^4 : |\sin(\theta)\hat{x}_1 - \cos(\theta)\hat{x}_2| \leq \epsilon\}.$$

- Jump set

$$\hat{\mathcal{J}} = \{\hat{x} \in \mathbb{R}^4 : |\sin(\theta)\hat{x}_1 - \cos(\theta)\hat{x}_2| \geq \epsilon\}.$$

with $0 < \epsilon < \mu$.

Simulation Scenario

Conclusion

- A hybrid observer is designed for the constrained state estimation of LTV systems subject to linear equality constraints
- Projection operator on surfaces is used to design the jump map of the observer
- Future work for nonlinear systems with nonlinear/inequality constraints

Conclusion

- A hybrid observer is designed for the constrained state estimation of LTV systems subject to linear equality constraints

- Projection operator on surfaces is used to design the jump map of the observer

- Future work for nonlinear systems with nonlinear/inequality constraints

Conclusion

- A hybrid observer is designed for the constrained state estimation of LTV systems subject to linear equality constraints
- Projection operator on surfaces is used to design the jump map of the observer
- Future work for nonlinear systems with nonlinear/inequality constraints

Conclusion

- A hybrid observer is designed for the constrained state estimation of LTV systems subject to linear equality constraints
- Projection operator on surfaces is used to design the jump map of the observer
- Future work for nonlinear systems with nonlinear/inequality constraints

Thank you!